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Building, Application and Realization of Quadratic Interpolation Polynomial with Lagrange Substrate

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Abstract

Interpolation is one of the important methods of function approximation, and it has been widely used. It was expounded in detail that the basic principles to construct quadratic interpolation polynomials with Lagrange substrate in this article, then it was applied to look-up table and evaluation, moreover they were realized by MATLAB7 programming; At last, the interpolation error was analyzed and estimated, and the interpolation function was contrasted with the original function. The results show that the Quadratic Lagrange interpolation was more accurate than that of the linear Lagrange interpolation.

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Keywords: Interpolation substrate; Interpolation polynomial; Lagrange interpolation; Interpolation errors

1. Introduction

In now life, Interpolation method is widely used, whether in daily industrial and agricultural production or defense advanced science and technology research, such as large and medium-sized electromechanical product optimization design, major project design, etc. Through measuring or experiment can only get finite discrete points in the interval of function values in $[a,b]$, it's only a function table. Only some functions have analytical expressions, but it's complex and not convenient to calculation. In order to research -

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the variation of function, often require to evaluate a list of function value in a given in [1]. So hopefully according to the given function table can get the simple function $P(x)$ reflects the characteristics of function $f(x)$, making $P(x)$ and $f(x)$ be approximate. With polynomial (including algebra, subsection and trigonometric polynomial) to approximating function is very effective numerical approximation tools, moreover Lagrange interpolation polynomials play an important role in numerical approach method.

2. Basic Property of Interpolation Method.

Basic properties of the interpolation polynomial are the existence and the uniqueness of the interpolation polynomials[2,7].

Supposing $P(x)$ is form of the interpolation polynomial like $P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$, using $H_n(x)$ represents all times is no more than n polynomial set. As a result $P(x) \in H_n(x)$, so-called the existence and unique of the interpolation polynomial $p(x)$, exactly representing collection of $H_n(x)$, existing only $p(x)$ meets $P_n(x_i) = y_i$ .Owing to $P_n(x_i) = y_i$, we can deduce formula (1):

$$\begin{cases} a_0 + a_1 x_0 + a_2 x_0{}^2 + \cdots + a_n x_0{}^n = y_0 \\ a_0 + a_1 x_1 + a_2 x_1{}^2 + \cdots + a_n x_1{}^n = y_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_0 + a_1 x_n + a_2 x_n{}^2 + \cdots + a_n x_n{}^n = y_n \end{cases} \tag{1}$$

Formula(1) is a $n+1$ member linear equation set about $a_0, a_1, \cdots a_n$ .If we want to proof the interpolation polynomial is exist and unique, we only need to proof equation set(1) exists unique solution, namely proofing the Coefficient determinant (2) of equation set(1) is not equal zero:

$$v_n(x_0, x_1, \cdots x_n) = \begin{vmatrix} 1 & x_0 & x_0{}^2 & \cdots & x_0{}^n \\ 1 & x_1 & x_1{}^2 & \cdots & x_1{}^n \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & x_n & x_n{}^2 & \cdots & x_n{}^n \end{vmatrix} \tag{2}$$

$v_n(x_0, x_1, \cdots x_n)$ is called Vandermonde determinant, using determinant attributes [3,8] is available:

$$v_n(x_0, x_1, \cdots x_n) = \prod_{i=1}^{n} \prod_{j=0}^{i-1} (x_i - x_j)$$

When $i \neq j$ and $x_i \neq x_j$, all factors meet $x_i - x_i \neq 0$, so $v_n(x_0, x_1, \cdots x_n) \neq 0$. Consequently, equation (1) only exists a set of solution $a_0, a_1, \cdots a_n$, because all the conclusion: if node $x_0, x_1, \cdots x_n$ is different, so meets existing and unique of n order of interpolation condition $P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$.

3. Lagrange's Interpolation

3.1. Basic idea

The basic idea of Lagrange interpolation is using nodes to construct following polynomial which is showed in formula (3) directly:

$$l_i(x) = \frac{\omega_{n+1}(x)}{(x-x_i)\omega'_{n+1}(x_i)} \tag{3}$$

Among them: $\omega_{n+1}(x) = (x-x_0)(x-x_1)\cdots(x-x_n)$

$$\omega'_{n+1}(x_i) = (x_i - x_0)\cdots(x_i - x_{i-1})(x_i - x_{i+1})\cdots(x_i - x_n) \tag{4}$$

It is easy to prove this polynomial has properties:

$$l_i(x_j) = \begin{cases} 0 & j \neq i \\ 1 & j = i \end{cases}$$

Therefore, n -order polynomial

$$L_n(x) = l_0(x)y_0 + l_1(x)y_1 + \cdots + l_n(x)y_n = \sum_{k=0}^{n} l_k(x)y_k \tag{5}$$

Must have properties:

$$L_n(x_i) = \sum_{k=0}^{n} l_k(x_i)y_k = l_i(x_i)y_i = y_i \qquad i = 0,1,\cdots,n$$

Namely, meeting condition of polynomial based on unique of interpolation polynomial, $L_n(x)$ is desired, consisted $L_n(x)$ of $l_i(x)(i=0,1,\cdots,n)$ called function of Lagrange's interpolation[4] .in fact, Lagrange's interpolation is $n+1$ the linear combination of basis functions, therefore combinatorial coefficient is the known function values of interpolation condition.

3.2. Error analysis

Principle of Lagrange interpolation: Supposing $L_n(x)$ is node of n times interpolation polynomials $x_0, x_1, x_2, \cdots, x_n$, in the interval of $[a,b]$, $L_n(x)$ and $f(x)$ is nearly, $f(x)$ is continuation in $[a,b]$. $f^{n+1}(x)$ Exists in $[a,b]$, Including $[a,b]$ contains node of $x_0, x_1, x_2, \cdots, x_n$ in any intervals, consequently, on any given $x \in [a,b]$, there always exists a node meeting $\xi \in (a,b)$ dependent on x make:

$$R_n(x) = f(x) - Ln(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\omega_{n+1}(x) \tag{6}$$

The remainder expression can be used only when the higher-order derivative of $f(x)$ is existing. ξ In the specific location (a,b) is usually impossible to be given, .if we can find out:

$$\max_{a<x<b} \left| f^{(n+1)}(x) \right| = M_{n+1}$$

Then, truncation error [5.6] of interpolation polynomial $P_n(x)$ approximating to $f(x)$:

$$|R_n(x)| \leq \frac{M_{n+1}}{(n+1)!}|\omega_{n+1}(x)| \tag{7}$$

3.3. Evaluated application

Through the following examples is to illustrate evaluated applications of the Lagrange interpolation method. For example: Value of a known function $y = \ln x$ is showed in Table1:

Table 1. Value of function $y = \ln x$

x	10	11	12	13	14
$y = \ln x$	2.3026	2.3979	2.4849	2.5649	2.6391

Using *Langrange* interpolation to solve the value of $\ln 11.75$

Resolving: in interpolation problem, due to reduce truncation errors, in choosing interpolation node, selecting some of the node is close to interpolation x .In this question, $x = 11.75$ is between 11 and 12 , When mapping linear interpolation problem should take node $x_0 = 11$ ， $x_1 = 12$,Based on equation (5):

$$L_1(x) = (x-12)/(11-12)\times 2.3979 + (x-11)/(12-11)\times 2.4849 = 0.087x + 1.4409$$

Setting $x = 11.75$ generation into, namely:

$$\ln 11.75 \approx L_1(11.75) \approx 2.4632$$

According to formula (7), there is: $|R_1(x)| \le \left(|\ln x|''\right)_\xi \Big/ 2!*|(11.75-11)(11.75-12)| \le 9.375\times 10^{-4}$

Similarly, in the quadratic, taking node $x_0 = 11, x_1 = 12, x_2 = 13$, according to formula (5):

Therefore, the approximation value of $\ln 11.75$ is:

$$\begin{aligned}\ln 11.75 \approx L_2(11.75) &= (11.75-12)\times(11.75-13)/(11-12)\times(11-13)\times 2.3979\\ &+(11.75-11)\times(11.75-13)/(12-11)\times(12-13)\times 2.4839\\ &+(11.75-11)\times(11.75-12)/(13-11)\times(13-12)\times 2.5649\\ &\approx 2.4638\end{aligned}$$

$$R_2(11.75) = \left(|\ln x|'''\right)_\xi \Big/ 3!*|(11.75-11)(11.75-12)(11.75-13)| \le 7.8125\times 10^{-5}$$

Through the check tachometer: $\ln 11.75 \approx 2.46385$

3.4. Matlab7 achievement

Based on the algorithm described by formula (5), we can write out the corresponding user-defined functions of Lagrange interpolation MatLab7 stored it in Lagrange. m file, its program code is shown in Fig.1.

```
function yh=lagrange(x,y,xh)
 n=length(x);
 m=length(xh);
 yh=zeros(1,m);
 c1=ones(n-1,1)
 c2=ones(1,m);
 for i=1:n
     xp=x([1:i-1,i+1:n]);
     yh=yh + y(i)*prod((c1*xh-xp'*c2)./(x(i)-xp'*c2));
  end
```

Fig. 1. user-defined functions source code of Lagrange interpolation Matlab7

```
X=[11    12]
Y=[2.3979   2.4849]
xh=[11.75]
yh=lagrange(X,Y,xh)
e1=log(xh)-yh
```

Fig. 2. linear Lagrange interpolation solving in MatLab7 program

Linear Lagrange interpolation solution of the MatLab7 is showed in Fig 2, save it in xxlglr.m file:

After the operation, we can get: $y_1 = 2.4632, e_1 = 7.0324\text{e-}004$

In the interval $[10,14]$, compare logarithmic curves and linear Lagrange interpolation curve in the MatLab7,Its operating results is shown in Fig 3.

Similarly, parabolic Lagrange interpolation solution of the MatLab7can is written, save it in eclglr.m file. After the operation get: $y_2 = 2.4639, e_2 = 4.6991\text{e-}005$.

In the $[10,14]$ interval, compared logarithmic curves and quadratic Lagrange interpolation curve with matlab7 is showed in Fig 4.

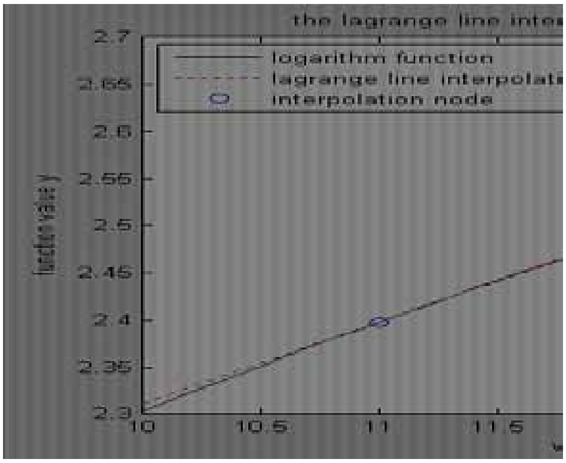

Fig. 3. Compared logarithmic curves and line Lagrange interpolation curve

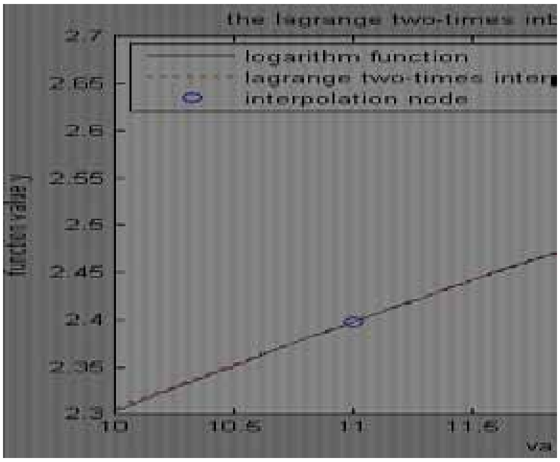

Fig. 4. Compared logarithmic curves and quadratic Lagrange interpolation curve

4. Conclusion

From the above analysis, we can know, when solution of $\ln 11.75$, the quadratic polynomial of the Language is smaller errors some than linear interpolation polynomials. In order to improve the precision, but then need to add nodes $l_i(x)$ all the change; also it is original basic function and cannot use, so waste of resources. To improving *lagrange* , When from low times to high times is calculated by using successive approximation has the calculated value is to simplify the calculation.

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References

[1] Li Qingyang, Wang Nengchao, Yi Dayi . *Numerical Analysis.*5th Ed [M], QingHua University publishing company, 2008.

[2] Huang Mingyou, Liu Bo, Xu Tao.*Numerical Calculation Method* [M], Science publishing company, 2005.

[3]Ma Zhiwei,Du Wei,Yan Xiaohong. *Full process teaching study and exercises full explain of High algebra*. 5th Ed [M] .Publishing Company of China age economy, 2009

[4] Staff room of calculating mathematics in Tongji University. *Modern Numerical Calculatio*n [M], Publishing Company of demos post and telecom, 2009

[5] Huang Duo,Chen Lanping,Wang Feng. *Numerical Analysis* [M], Science publishing company, 2003

[6] Li Qingyang, Guan Zhi, Bai Fengbin.mathematics calculation principle[M],QingHua University publishing company, 2003

[7] Xue Yi. *Numerical Analysis and experiment* [M], publishing company of Beijing Industrial University, 2006

[8] Feng Jianhu, Ju Gangming,Lie Yufeng. Numerical Analysis [M], Science publishing company, 2004